Ratchet-Like Solitonic Transport in Quantum Hall Bilayers

Ramaz Khomeriki[1],* Lasha Tkeshelashvili[2,3],† Tinatin Buishvili[1], and Shota Revishvili[1]
[1)] Department of Physics, Tbilisi State University, Chavchavadze ave. 3, Tbilisi 0128, GEORGIA
[2)] Institut für Theorie der Kondensierten Materie, University of Karlsruhe, P.O. Box 6980, 76128 Karlsruhe GERMANY; and
[3)] Institute of Physics, Tamarashvili str. 6, Tbilisi 0177, GEORGIA

The pseudo-spin model for double layer quantum Hall system with total landau level filling factor $\nu = 1$ is discussed. Unlike the "traditional" one where interlayer voltage enters as static magnetic field along pseudo- spin hard axis, in our model we consider applied interlayer voltage as a frequency of precessing pseudo-magnetic field lying into the easy plane. It is shown that a Landau-Lifshitz equation for the considered pseudo magnetic system well describes existing experimental data. Besides that, the mentioned model predicts novel directed intra-layer transport phenomenon in the system: unidirectional intra-layer energy transport is realized due to interlayer voltage induced motion of topological kinks. This effect could be observed experimentally detecting counter-propagating intra-layer inhomogeneous charge currents which are proportional to the interlayer voltage and total topological charge of the pseudo-spin system.

PACS numbers: 73.43.Lp, 05.45.Yv

In the recent years there exists a steady interest in quantum Hall bilayers with total Landau level filling factor $\nu = 1$ due to their anomalous transport and tunneling properties [1]. Quantum Hall bilayers consist of electrons confined in closely separated two dimensional semiconductor layers in high magnetic fields. In the absence of interlayer voltage, each layer of the system has a filling factor $\nu_1 = \nu_2 = 1/2$. Because of the layers are identical, an electron in one layer could be identified with a fake spin up, while in the other layer with a pseudo-spin down. Therefore, the system can be phenomenologically described via the pseudo-spin formalism [2]. The z component of the overall pseudo-spin vector specifies the charge imbalance between the layers. It is clear that the system has a lower energy when the pseudo-spin points neither up nor down, but rather lies in the plane, reflecting the fact that in the ground state electrons are equally distributed between the two layers. Therefore, within this formalism, double layer quantum Hall system is treated as an easy plane ferromagnet with a hard axis anisotropy and an electron tunneling between the layers corresponds to the spin flips in pseudo-spin language.

This is a quite satisfactory model for the isolated double layer systems. However, problems start to arise as soon as one considers a real experimental situation with applied interlayer voltage and induced tunneling current. Traditionally, in the phenomenological Hamiltonian, interlayer dc voltage is interpreted as a constant magnetic field along z axis [2]. Although this model correctly describes the physics at low interlayer voltages [3], it fails to describe experimentally observed current-voltage characteristics [1] for large interlayer voltages. Besides that, in order to capture the essential physics of the system, one has to introduce effective damping mechanism in the formalism.

Here we use a different interpretation of the interlayer voltage, comprehensive analysis of which can be found in Ref. [4]. Particularly, we treat the interlayer voltage as a circularly polarized oscillating magnetic field with the magnitude equal to the tunneling amplitude. In addition, it is important that the driving magnetic field has the frequency proportional to the interlayer voltage itself. In the absence of damping, the traditional model coincides to our's one in the rotating (with the frequency of oscillating field) reference frame. Nevertheless, after introducing the damping, the ground state of the system in these two models differ from each other (spins aligned in easy-plane in our model and tilted from easy-plane in the traditional one) and consequently the equations of motion are no more invariant with respect of changing the reference frame.

In the equation of motion we choose Landau-Lifshitz damping term since it is most convenient for the systems with easy-plane symmetries. The point is that it does not lead to the damping to a definite direction in easy-plane. Therefore, it is not needed to break the symmetry "by hand" [3]. This would not be justified, since the only physically measurable quantity is the charge imbalance between the layers, i.e. z component of pseudo-spin, while all other components of pseudo spin are not physically measurable. As we shall demonstrate below, our model well describes experimental observations by Spielman et.al. [1]. Moreover, this model leads to the prediction of directed solitonic transport phenomenon, which can be directly verified on the experiment. Note that, this novel effect appears as a natural consequence of the identification of interlayer voltage with an ac driving magnetic field. Indeed, as it had been suggested recently by Flach and co-workers in Ref. [5], ac driving force with a zero mean value may cause a directed transport in a nonlinear system. In addition, it was shown that the necessary condition for this phenomenon to appear is that

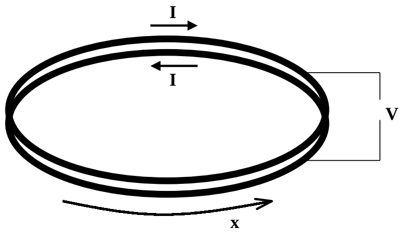

FIG. 1: Suggested experimental setup for observation of inhomogeneous counter propagating intra-layer charge currents by applying an interlayer dc voltage on a quantum Hall bilayer.

the system is characterized by nonzero total topological charge. As is well established, such topological picture may exist only if the ground state of a system is degenerated, what is the case in quantum Hall bilayers according to our model.

Here we present the symmetry analysis [5] and numerical simulations in order to show that the directed inhomogeneous intra-layer current appears due to propagation of topological excitations in a quantum Hall bilayer. Realistic experimental setup is suggested in order to observe this exotic phenomenon.

The Model: The effective Hamiltonian density of our phenomenological model for double layer quantum Hall (pseudo) ferromagnet is given by:

$$\mathcal{H} = \frac{\rho_E}{2}\left|\frac{\partial m_+}{\partial x}\right|^2 + \beta\big(m_z\big)^2 - \frac{\Delta_{SAS}}{2}\left\{m_+ e^{i\omega t} + \text{c.c.}\right\} \quad (1)$$

where $\vec{m}(x,t)$ is a order parameter unit vector; $m_\pm = m_x \pm i m_y$ ($m_z = \nu_1 - \nu_2$ describes local electrical charge imbalance between two layers); ρ_E is the in-plane spin stiffness, β gives a hard axis anisotropy, Δ_{SAS} is a tunneling amplitude, $\omega = eV/\hbar$ includes interlayer voltage V, e is an electron charge and "c.c." indicates a complex conjugated term. Then Landau-Lifshitz equations of motion, which conserve the length of a local spin density, can be presented as follows:

$$\frac{\partial \vec{m}}{\partial t} = \left[\vec{m} \times \vec{H}\right] - \gamma\left[\vec{m} \times \left[\vec{m} \times \vec{H}\right]\right], \quad (2)$$

where

$$\vec{H} = -2\left\{\frac{\partial \mathcal{H}}{\partial \vec{m}} - \frac{\partial}{\partial x}\left[\frac{\partial \mathcal{H}}{\partial \frac{\partial \vec{m}}{\partial x}}\right]\right\}, \quad (3)$$

is the effective magnetic field and γ is the Landau-Lifshitz dimensionless damping coefficient.

It is easy to see that in the absence of damping our model coincides with the traditional one [2]. Indeed, by redefining the order parameter as $m_+ \to m_+ \exp(-i\omega t)$, the same equations of motion (2) could be derived from the well known Hamiltonian density:

$$\mathcal{H} = \frac{\omega}{2} m_z + \frac{\rho_E}{2}\left|\frac{\partial m_+}{\partial x}\right|^2 + \beta\big(m_z\big)^2 - \Delta_{SAS} m_x. \quad (4)$$

Therefore in case of zero damping the Hamiltonians (1) and (4) describe the same physics in different frames. However, for nonzero damping these two approaches are no more equivalent.

Numerical Experiment and Analytical Solutions: To specify the problem, we consider a quasi-one dimensional quantum Hall double layer system with the periodic boundary conditions (quantum Hall "ring" as is shown in Fig. 1). By applying the appropriate discretization procedure $\partial\vec{m}(x,t)/\partial x \to \vec{m}_{i+1}(t) - \vec{m}_i(t)$ and $\partial^2\vec{m}(x,t)/\partial x^2 \to \vec{m}_{i+1}(t) + \vec{m}_{i-1}(t) - 2\vec{m}_i(t)$, the equations of motion (2) reduce to a set of $3N$ ordinary differential equations (N is a number of discretization points). We solve this set of equations using MATLAB. The results are presented in Fig. 2. One can see that the charge imbalance m_z is very small even for very large ω-s (comparable with β). In the case of small voltages, as is expected, z component of the order parameter is slightly tilted from the easy-plane (the upper panel in Fig. 2), while for the large voltages, which corresponds to the

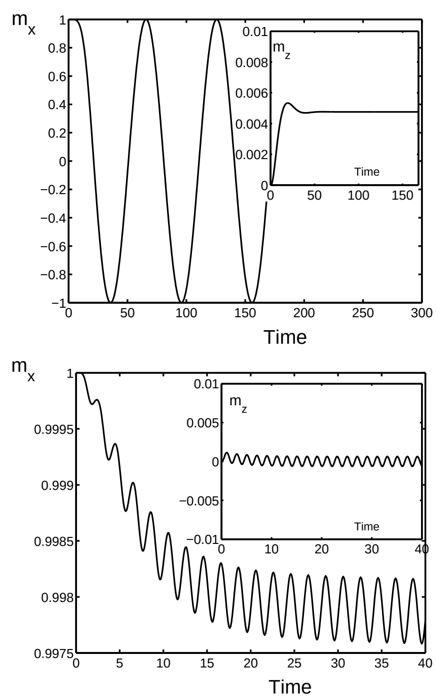

FIG. 2: Numerical simulation results: pseudo-spin evolution for low (upper graph) and large (lower graph) interlayer voltages. Main plots and insets represent pseudo-spin x and z components, respectively. Note that m_z is small in both low and large voltage limits. In numerical experiments it is taken $\beta = 10$, $\Delta_{SAS} = 0.002$, $\gamma = 0.01$, $\omega = 0.1$ (upper graph) and $\omega = 3$ (lower graph).

large driving frequencies, z component of the order parameter oscillates around the zero value (the lower panel in Fig. 2). Such behavior for the large voltages follows from the fact that the driving frequency is proportional to the voltage. As is well known [6] a high frequency driving field, even it has a large amplitude, can cause only small oscillations in a system around the equilibrium point.

These observations suggest to simplify further the equations of motion. Particularly, in the limit $m_z \to 0$, one can rewrite Eq. (2) as follows:

$$4\beta\gamma m_z + \frac{\partial m_z}{\partial t} = 2\rho_E \frac{\partial^2 \varphi}{\partial x^2} - 2\Delta_{SAS}\sin(\varphi + \omega t) \quad (5)$$

$$\frac{\partial \varphi}{\partial t} = 4\beta m_z, \quad (6)$$

where the phase variable φ is defined from the relation $m_+ = \sqrt{1-m_z^2}\exp(i\varphi)$. Reminding that m_z describes a local charge imbalance between the layers, Eq. (5) could be interpreted as the charge continuity equation with a damping term, where

$$J_S = -2e\rho_E \frac{\partial \varphi}{\partial x}, \quad J_{tun} = 2e\Delta_{SAS}\sin(\varphi + \omega t) \quad (7)$$

are intra-layer current in each layer and interlayer tunneling current, respectively. Further, substituting (6) into the continuity equation (5), finally we obtain:

$$4\beta\gamma \frac{\partial \varphi}{\partial t} + \frac{\partial^2 \varphi}{\partial t^2} - 8\rho_E \beta \frac{\partial^2 \varphi}{\partial x^2} + 8\beta\Delta_{SAS}\sin(\varphi + \omega t) = 0. \quad (8)$$

For the first time, without the gradient term, this equation was suggested by Wen and Zee [7] (see also Ref. [8]). The heuristical considerations in Ref. [7] as well as our numerical simulations of Eq. (2) suggest that, there are completely different regimes of the pseudo-spin dynamics for the low ($eV/\hbar \ll \sqrt{8\beta\Delta_{SAS}}$) and large ($eV/\hbar \gg \sqrt{8\beta\Delta_{SAS}}$) voltages. In particular, in the first case under the periodic boundary conditions the analytical solution of (8) should be sought as homogeneous time rotations around z axis with the angular frequency $\omega = eV/\hbar$:

$$\varphi = \phi_0 - \omega t, \quad (9)$$

where ϕ_0 is a constant quantity. Substituting (9) into the motion equation (8) one can define ϕ_0 as follows:

$$2\Delta_{SAS}\sin\phi_0 = \omega\gamma. \quad (10)$$

Moreover, from (7) the expression for the tunneling current reads:

$$J_{tun} = (e^2/\hbar)V\gamma. \quad (11)$$

For the case of large voltages the solution is sought as homogeneous small time oscillations around a definite direction in the easy plane as:

$$\varphi = A\sin(\phi_0 + \omega t), \qquad A \ll 1, \quad (12)$$

where the constants A and ϕ_0 must be defined perturbatively by substituting (12) into the equation of motion (8) (see Refs. [8, 9]). As a result one obtains:

$$\tan\phi_0 = \frac{4\beta\gamma}{\omega}, \qquad A = \frac{8\beta\gamma}{\omega\sqrt{\omega^2 + 16\beta^2\gamma^2}}, \quad (13)$$

and the dc component of tunneling current is given by:

$$J_{tun} = 2e\Delta_{SAS}\overline{\sin\Big[A\sin(\phi_0 + \omega t) + \omega t\Big]} =$$
$$e\Delta_{SAS}A\sin\phi_0 = \frac{e^2}{\hbar}V\frac{32\Delta_{SAS}^2\beta^2\gamma}{(eV/\hbar)^2\Big[(eV/\hbar)^2 + 16\beta^2\gamma^2\Big]}. \quad (14)$$

It is easy to see that Eqs. (11) and (14) qualitatively well describe the experimentally observed tunneling current voltage characteristics [1]. Indeed, at the low voltages the current increases with voltage, while for the large voltages the tunneling current decreases as $1/V^3$.

In the consideration given above it was assumed that total topological charge of the system is zero. Consequently we obtain that the intra-layer current is zero for any voltages according to the definitions (7). The situation, however, drastically changes if nonzero topological charge is present in the system.

Directed intra-layer transport: In order to apply the symmetry analysis we should write down the expression for the density of energy flux in the system (see e.g. Ref. [5]):

$$J_E = -\rho_E \frac{\partial \varphi}{\partial x}\frac{\partial \varphi}{\partial t} \quad (15)$$

and define topological charge of the system as follows:

$$Q = \Big[\varphi(+\infty) - \varphi(-\infty)\Big]/2\pi. \quad (16)$$

In general, the topological charge Q may take any integer value. For simplicity in our numerical experiment we choose an initial topological charge as $Q = 1$. That is, for the discretized problem of N "spins" we take:

$$m_+^j = exp\Big[2i\pi(j-1)/(N+1)\Big], \qquad m_z^j = 0, \quad (17)$$

and apply the periodic boundary conditions $\vec{m}^1 = \vec{m}^{N+1}$ to the system. According to the general approach [5] let us consider the following symmetry transformations:

$$x \to -x, \qquad \varphi \to -\varphi. \quad (18)$$

These symmetry transformations leave the topological charge (16) invariant but change the sign of the density of energy flux (15). In this case the symmetry properties of equations of motion becomes crucial. If they are invariant under the symmetry transformations (18) then the averaged energy flux in the system is zero, otherwise directed energy flux exists even in presence of noise only

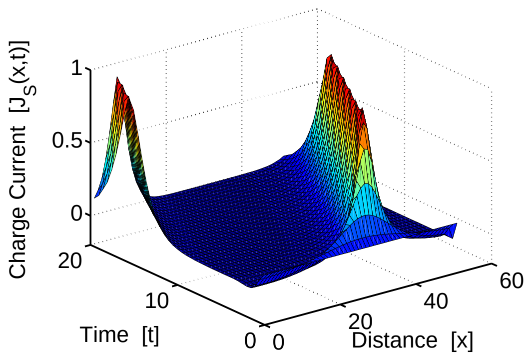

FIG. 3: Distribution of the intra-layer local charge current versus time and distance in the case of nonzero initial topological charge ($Q = 1$) and nonzero interlayer voltage. In the numerical experiments the periodic boundary conditions are used.

[5]. The equations of motion (8) are not invariant with respect the symmetry transform (18) and therefore, the directed intra-layer energy transport is expected for any applied nonzero interlayer voltage. Similarly, for zero voltage Eq. (8) is invariant under the symmetry transformation (18) and thus averaged energy flux should be zero.

The numerical analysis of both, the initial motion equation (2) and its reduced version (8) completely agrees with the predictions of the symmetry analysis. Thus, in the presence of nonzero total topological charge in the system, directed energy transport should be observed. Moreover, the direction of the transport can be changed just by inverting the sign of the voltage. Besides that, we can further extend the analytical consideration noting that the equation of motion (8) in the " moving" reference frame $\varphi \to \varphi - \omega t$ is nothing but dc driven-damped sine-Gordon equation (see e.g. Refs. [10, 11]). The role of the dc driven force plays the term $f = 4\beta\gamma\omega$. As is well known, sine-Gordon equation in the absence of driving force supports solitary wave solutions with nonzero topological charge. these solutions often are termed as kinks and are given by the following expression:

$$\varphi = 4 \arctan \left\{ \exp \left[\sqrt{\frac{\Delta_{SAS}}{\rho_E}} (x - x_0) \right] \right\}. \tag{19}$$

Applied driving force results in the motion of that object, velocity of which will be proportional to the driving force (applied voltage). Thus, by increasing of the voltage, the intra-layer energy transport (and consequently the inhomogeneous charge current) will be increased.

It should be especially mentioned that if one would use the Hamiltonian (4) as a starting point of the analysis, one gets the result similar to Eq. (8) but with $\omega = 0$. Consequently, no directed motion is possible according to the traditional model.

Conclusions: The driven-damped pseudo-magnetic model is elaborated in order to describe the dynamics in quantum Hall bilayers at the total filling factor $\nu = 1$. The symmetry analysis and numerical simulations are used in order to show that the directed transport exists in the system in the presence of nonzero topological charge. Realistic experimental setup is suggested for observing the suggested phenomenon. Initial nontrivial topological charge [like presented in Exp. (17)] in the system could be realized in the laboratory experiments by application of a weak in-plane magnetic field along the double layer "ring" (x direction in Fig. 1), for which a commensurate pseudo-spin distribution appears. Then, switching off the in-plane field and applying an interlayer dc voltage it will be possible to observe the inhomogeneous counter propagating currents in each layer (see Fig. 3). It is obvious that the thermal fluctuations will decrease the topological charge in a double layer system (like it happens in narrow superconducting channels [12]) and as a result the intra-layer current eventually should decrease as well.

Acknowledgements: R. Kh. is indebted to Ramin Abolfath and Kieran Mullen for stimulating discussions and is supported by USA Civilian Research and Development Foundation award No GP2-2311-TB-02 and NATO reintegration grant No PST.EV.979337. L.T. acknowledges financial support from the Deutsche Forschungsgemeinschaft (DFG) under Bu 1107/2-2 (Emmy-Noether program).

* khomeriki@hotmail.com
† lasha@tkm.physik.uni-karlsruhe.de

[1] I. B. Spielman et al., Phys. Rev. Lett., **84**, 5808, (2000);
[2] K. Moon *et al.*, Phys. Rev. B, **51**, 5138, (1995); K. Yang *et al.*, Phys. Rev. B, **54**, 11644, (1996).
[3] Y. N. Joglekar and A. H. MacDonald, Phys. Rev. Lett., **87**, 196802, (2001).
[4] M. Abolfath, R. Khomeriki and K. Mullen, Phys. Rev. B, **69** (April 15 issue), (2004).
[5] S. Flach et al., Phys. Rev. Lett., **88**, 184101, (2002); M. Salerno and Y. Zolotaryuk, Phys. Rev. E, **65**, 056603, (2002).
[6] L. D. Landau and E. M. Lifshitz, *Mechanics*, Pergamon Press, Oxford (1976).
[7] X.-G. Wen, and A. Zee, Phys. Rev. B., **47**, 2265 (1993).
[8] M. M. Fogler and F. Wilczek, Phys. Rev. Lett., **86**, 1833, (2001).
[9] R. E. Eck, D. J. Scalapino and B. N. Taylor, Phys. Rev. Lett., **13**, 15, (1964).
[10] B. A. Malomed, Phys. Rev. B., **43**, 10197, (1991).
[11] O. M. Braun et al., Phys. Rev. B., **67**, 066602, (2003).
[12] J. S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498, (1967)